Quiz #2

Toggerson Physics 564

Instructions

Answer each question in a few words or a few lines of algebra.
If you are doing a lot of work, you are going down the wrong path.

No resources.

Question 1 [5pts]

Write down the S_z operator in the S_z basis. That's it, no need to explain or anything!

(If this feels too simple, you are actually probably doing it right!).

In the S_z basis, the S_z operator will simply be the eigenvalues on the diagonal, so

$$\boldsymbol{S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}$$

Question 2 [8pts]

Suppose a spin-1/2 particle, in the S_z basis, is in the state

$$|\chi\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix}$$

What is the probability of getting $+\hbar/2$ if you measure S_z?

From the operator above, we can surmise that the upper component of this spinor corresponds to the $S_z = +\hbar/2$ eigenvalue,

$$|\chi\rangle = \frac{1+i}{\sqrt{6}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{2}{\sqrt{6}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$|\chi\rangle = \frac{1+i}{\sqrt{6}}\left|S_z = +\frac{\hbar}{2}\right\rangle + \frac{2}{\sqrt{6}}\left|S_z = -\frac{\hbar}{2}\right\rangle$$

which is commonly written as

$$|\chi\rangle = \frac{1+i}{\sqrt{6}}|\uparrow\rangle + \frac{2}{\sqrt{6}}|\downarrow\rangle$$

Thus,

$$P = \left|\frac{1}{\sqrt{6}}(1+i)\right|^2$$
$$P = \frac{1}{6}(1+i)(1-i)$$
$$P = \frac{1}{6}(1+1) = \frac{2}{6} = \frac{1}{3}$$