What about other values of angular momentum? ½ perhaps?

Spin ½ Objects need a 2-rep (not the 3 we've been working with)

Fortunately the Lie Algebra helps us out here

$$R(\varphi) = \exp\left[\frac{-i\varphi}{\hbar}\vec{S}\cdot\hat{n}\right]$$

where $\vec{S}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ for example and, in general, $S_i = \hbar/2\,\sigma_i$

$$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \qquad \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

Since these obey the same algebra, all good!

Or are we?

$$R(\varphi) = \exp\left[\frac{i}{\cancel{\hbar}}\varphi\frac{\cancel{\hbar}}{2}\vec{\sigma}\cdot\hat{n}\right] = \exp\left[i\left(\frac{\varphi}{2}\right)\vec{\sigma}\cdot\hat{n}\right]$$

I need to go 720° to get back to where I started!

Demo of Dirac belt trick.

SU(2) and Representations

The Pauli matricies are actually generators of $SU(2)$
- The group of all 2×2 unitary matricies of det $=1$
- Also 3-parameters!

$$\left.\begin{array}{l} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow 4 \\ \rightarrow ad - bc = 1 \end{array}\right\} \begin{array}{l} (n^2 \text{ generally}) \\ 4-1=3 \end{array}$$

- I can make any $SU(2)$ matrix by $e^{-i\alpha_j \sigma^j}$

The fact that the number of parameters is the same for $SU(2)$ & $SO(3)$ is why they can have the same Lie algebra: $SU(2)$ is isomorphic to $SO(3)$

In fact locally, we can say that the rotation matricies we've been dealing with for vectors are the 3-rep of $SU(2)$. The 2-rep is called the fundamental rep

Spin $3/2$ would be in the 4-rep!

Generate the Lie Algebra of the correct dimensionality, and you can see how anything transforms under rotations!

Beyond that, nature really likes $SU(2)$
- Nuclear physics: p^+ & n^0 are almost a spin "up" & "down"
- The weak force is in a 3-rep of $SU(2)$
- ...