

Quiz #1

Toggerson Physics 564

Instructions

Answer each question in a few words or a few lines of algebra.

If you are doing a lot of work, you are going down the wrong path.

No resources.

Question 1 [8pts]

If

$$H \Psi(x, t) = i\hbar \frac{\partial}{\partial t} \Psi(x, t)$$

we can define a *time translation operator* $U(t)$ which satisfies

$$U(t) \Psi(x, 0) = \Psi(x, t)$$

What is $U(t)$ and give a short description of how we derived it?

Note: I am looking for the final form of $U(t)$ exactly: I will give slack on negative signs, but do be sure your units are correct! As for the “derivation” you do not need to reproduce it mathematically line-for-line; a few words on the general process will be sufficient.

$$U(t) = \exp\left[-\frac{it}{\hbar} H\right]$$

Which we obtained by Taylor expanding $\Psi(x, t)$ about $t = 0$, factoring to the right $\Psi(x, 0)$ and then recognizing the infinite sum

$$\sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{i}{\hbar} H t\right)^n = \exp\left[-\frac{it}{\hbar} H\right]$$