13 Feb **Application of Discrete Groups - Parity** 18

Slide

A.Y.A.C.I. · Get ready for your quiz on 6.4. No resources during quiz. Plenty of room to write.

Objectives

By the end of today's class, you should be able to...

- Expand the conceptual understanding of parity from your reading into the Z_2 group including:
  - -) Creating a multiplication table
  - -) Determine the group's characteristics: Abelian & Unitary
  - -) Construct the following representations:
    - * Trivial rep
    - * Non-trivial 1-rep
    - * The 4-rep which acts on 4-vectors
- Define unitarity (homework!) and why useful
- List & define other groups common in physics: $U(n)$, $SU(n)$, $O(n)$, $SO(n)$ ✓
- Define scalar, pseudoscalar, vector, pseudovector and give examples of each.
- Determine if a Hamiltonian commutes with the parity operator.
- Describe the impacts to the wave function if $[H, \Pi] = 0$
- Explain the concept behind a multipole expansion.

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Parity as a group

In your reading, you were exposed to the idea of parity: $\Pi \psi(x) = \psi(-x)$.

Now, I want you to combine that with our discussion on groups.

Parity makes an excellent example of the application of discrete groups to physics. (We'll get to continuous groups soon enough!)

Determine for Π — Slid Ex

- The multiplication table: $\begin{array}{c|cc} & I & \Pi \\ \hline I & I & \Pi \\ \Pi & \Pi & I \end{array}$
- The trivial rep: All 1's
- The non-trivial 1-rep: $I = 1$, $\Pi = -1$
- The 4-rep which acts on 4 vectors like (t, x, y, z) $\left(\begin{smallmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{smallmatrix}\right)$ ← Maybe easiest!
- Is it Abelian?

Another important property of the parity (Z_2) group - Unitarity

If we look at the 4-rep, we can see explicitly that

$$\Pi^{\dagger} = \Pi^{*T} = \Pi^{-1} \quad (= \Pi \text{ in this case})$$

Since $\Pi^{\dagger} = \Pi^{-1}$, we say Z_2 is unitary.

Unitary groups, called $\mathcal{U}(n)$, are some of the most common & important in physics

- The SM is $SU(3) \times SU(2) \times \boxed{U(1)}$

Why are unitary groups so common/important? Ex
Unitary transformations of operators $U^{+} O U$ preserve orthonormality of eigenvectors!

$\mathcal{U}(n)$, then is the group of $n \times n$ matricies w/ $\mathcal{U}^{+}\mathcal{U} = 1$

You will prove these are groups in your homework.

Some other groups worth knowing due to their importance to physics (we'll revisit some of these)

$\mathcal{U}(n)$: $n \times n$ matricies w/ $\mathcal{U}^{+}\mathcal{U} = 1$
$S\mathcal{U}(n)$: special unitary group - $n \times n$ matricies w/ $M^{+}M = 1$ and det 1
$O(n)$: orthogonal group - $n \times n$ real matricies | $O^{+}O = O^{T}O = 1$
$SO(n)$: special orthogonal group — $M^{T}M = 1$ and det 1

Sh

What is Π's 1-rep? Ex
$O(1)$

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How do different quantities transform?

We know that the two vectors $\{\vec{x}, \vec{p}\} \in \vec{v}$ switch sign under parity

$$\Pi^{+} \vec{v}\, \Pi = -\vec{v}$$

We say that these vectors are odd under parity

What about these other quantities

1) Scalars (like mass) Even
2) Quantities arising from cross products (like $\vec{L}$ or $\vec{B}$) Even pseudovectors
3) Quantities of the form $\vec{a} \cdot (\vec{b} \times \vec{c})$ such as $\Phi_B = \int \left(\frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{|r|^3} \right) \cdot d\vec{a}$

Even pseudoscalars

Summary

Why this matters in general

You know that $\hat{T}$ is even (a "true" scalar)

While you will prove in your next HW, it should come as no suprise that in this case $[T, H] = 0$

ABCI

Ex

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Slide ABCD Ex

If V is spherically symmetric (i.e. QMHO or hydrogen) what is V?
A "true" Scalar

If both $[T, \pi] = 0 = [V, \pi] \Rightarrow [H, \pi] = 0$

↳ Eigenstates of H must also & simultaneously be eigenstates of π

Skels Ex

- QMHO
- Classical gravity : $V = GM/r^2$
  - → Ball in mirror demo

Demo

- E&M
- Strong force
- Not weak!