Need: iOLab, Mag, [assumption] side board

6 Feb Finish Review & Start Symmetries 4

A.Y.A.C.I

Give ~15 min to work on packet followed by a quick debrief Time

Goal & Objectives

By the end of this class, you should be able to...
- Describe the differences between simple surface-level symmetries & underlying symmetries in the equations of motion.
- Recognize that patterns and degeneracies are often the result of an underlying symmetry.
- Identify a symmetry underlying a pattern or degeneracy
- Given a Lagrangian w/ a symmetry, determine the corresponding conserved quantity
- State Noether's Theorem and give a few examples.

Where I am pulling from

For this discussion of symmetries and the related topic of group theory, I am pulling from several places
- Griffiths, Chapter 6
- Griffiths Intro to Elementary Particles, Ch 4
- Matthews & Walker 2nd Ed., Ch 16
- Marion & Thorton Classical Dynamics, Ch 7

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Motivation: word cloud Slid

Reviewing Mathematical Symmetries

Show $f(x) = e^{-x^2} \sin(x^3)$ Slid

- Give me a symmetry Ex
  - It's odd
  - Animate Slid
- How are the following related? Ex
  - $[f(-x)]^6 \overset{?}{:} [f(x)]^6 =$
  - $\left.\frac{\partial f}{\partial x}\right|_{+2} \overset{?}{} \left.\frac{\partial f}{\partial x}\right|_{-2} =$
  - $\int_{-3}^{+3} f(x)\,dx = 0$

From the symmetry of the function, we have learned a lot!

Introduction to Symmetries in Physics

Physics is often the reverse: Slid

1) We find a neat pattern:
   a) The 1^{st} excited states of a particle in a 2-D box are two-fold degenerate:
      $n_x = 1,\ n_y = 2$ vs. $n_x = 2,\ n_y = 1$
   b) Planets return to the same spot each "year"
2) These patterns are then found to be due to some underlying symmetry
   a) The 2-D box is the same if I rotate by 90°

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b) Newton's Law of Gravitation (& GR for that matter)

$$F = -\frac{GMm}{r^2}$$

is spherically symmetric: Invariant under rotations

This planetary orbit example is particularly important as it shows us the types of symmetries most important for physics: Slid

- The ancient Greeks thought that the planets __must__ move in circles as that is the most symmetric shape
  - → Obviously wrong: ellipses
- Newton discovered that, while the paths themselves are not spherically symmetric, the underlying E.O.M. are:

$$m\ddot{r} = -\frac{GmM}{r^2}$$

Interesting Aside: Did Newton use the fact that the gravitational force "ought" to be spherically symmetric (effectively following Aristotle & co.) to figure out GMm/r^2? What if I hadn't been?

w.r.t. : with respect to

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Going Deeper: Some "Dumb" Patterns

Let's do some "dumb" experiments:
- Drop an ioLab & time the fall
  - → Different times
  - → Different places
  - → Different orientations

$\left.\begin{matrix}\text{Different times}\\ \text{Different places}\\ \text{Different orientations}\end{matrix}\right\}$ Same Results!

Just like how the planets return to their original positions, we notice a *symmetry*
- The laws of physics seem invariant w.r.t. translations in time
- & space
- & orientation

Are these symmetries hinting at something deeper like how the closed orbits of planets ~~begrite~~ indicate the spherical symmetry of GMm/r^2?

Let's look at spatial invariance

Eg

Consider a Lagrangian in rectangular coordinates

$$L(x_i, \dot{x}_i, t)$$

(Following M&T 7.9 here)

We will only consider
- Closed systems $\partial L/\partial t = 0$
- Speed-independent potentials $\partial U/\partial \dot{x}_i = 0 \;\forall\, \dot{x}_i$

Assum
Side
boar

$\therefore \rightarrow$ Therefore

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E.g. the ioLab's Lagrangian

$$L = \frac{1}{2} m\dot{y} - mgy$$

Let's shift our entire system an infintesimal amount:

$$x_i \rightarrow x_i + \delta x_i$$

The change in L, δL, is (chain rule)

$$\delta L = \frac{\partial L}{\partial x_i} \delta x_i + \frac{\partial L}{\partial \dot{x}_i} \delta \dot{x}_i$$

(Einstein convention of sum over i implied)

What is $\delta \dot{x}_i$? Ex

Zero: all we did was slide our origin which does not change $\dot{x} = \frac{dx}{dt}$

$$\therefore \quad \delta L = \frac{\partial L}{\partial x_i} \delta x_i$$

Recall the principle of least action: Mug

- $S \equiv \exp[\int L\, dt] \leftarrow$ Action &
- Particles follow the path of least action Assump
  - -) Minimizing $S \rightarrow \delta S = 0 \rightarrow \delta L = 0$ Side
  - -) This is what you use to derive the Lagrangian e.o.m. $\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_i} = \frac{\partial L}{\partial x_i}$ Boar

$\downarrow$

$\because \rightarrow$ because

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In order for $\delta L = 0$,

$$\frac{\partial L}{\partial x_i} = 0 \quad \forall \ x_i$$

Coming back to the Euler-Lagrange E.O.M.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} = \frac{\partial L}{\partial x_i}$$

$$\frac{\partial L}{\partial x_i} = 0 \rightarrow \frac{\partial L}{\partial \dot{x}_i} = \text{constant}$$

Now, putting in $L = T - U$ recalling
- $T = \frac{1}{2} m \dot{x}_i^2$
- $\partial U / \partial \dot{x}_i = 0$

$$\frac{\partial L}{\partial \dot{x}_i} = \text{constant}$$

$0 \because \partial U / \partial \dot{x}_i = 0$

$$\frac{\partial}{\partial \dot{x}_i}(T - \cancel{U}) = \text{constant}$$

$$\frac{\partial}{\partial \dot{x}_i}\left(\frac{1}{2} m \dot{x}_i\right) = \text{constant}$$

$$\sum_i m x_i = \text{constant} \leftarrow \text{Momentum is conserved}$$

$\therefore$ A translational symmetry in space $\rightarrow$ momentum conservation

Noether's Theorem

Generalizes what we did!

Symmetry	Conserved Quantity
Space Translation	Momentum
Time Translation	Energy
Orientation / Rotation	Angular Momentum (& Boosts)
EM Gauge $\phi \to \phi + \lambda(x)$	Electric Charge

Slide

Einstein's Obit

Slide