Looking @ Ψ, $\exists$ 2 options a + & a −!

Fermions must be − (antisymmetric)
Bosons must be + (symmetric)
} Under all interchanges: space, spin, ...

Slide SM

Also $^3He^*$ & $^4He^*$

Eg Two electrons, both in a H 1s state
· Start w/ space:

$$\psi = \psi_{1s}(r_1)\,\chi_1\,\psi_{1s}(r_2)\,\chi_2 \pm \psi_{1s}(r_1)\chi_?\,\psi_{1s}(r_2)$$

Spin — To be figured out

We see that this is symmetric in space (changing $r_1 \to r_2$ doesn't do anything, just a variable redefinition!)

Spin part must be antisymmetric

$$\psi = \psi_{1s}(r_1)\,\psi_{1s}(r_2)\left[|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right]$$

Must have ℓ & $m = 0$ as all $\ell = 1$ states are symmetric

Slide Other problems

1) $\psi = \exp(ip_a r_1/\hbar)\exp(ip_b r_2/\hbar) + \exp(ip_b r_1/\hbar)\exp(ip_a r_2/\hbar)$

2) $|\psi\rangle = |1s\rangle_{r_1}|\uparrow\rangle|2s\rangle_{r_2}|\uparrow\rangle - |2s\rangle_{r_1}|\uparrow\rangle|1s\rangle_{r_2}|\uparrow\rangle = (\text{Asym space})\,|\uparrow\uparrow\rangle$

3) $|1s\rangle_{r_1}|\uparrow\rangle|1s\rangle_{r_2}|\uparrow\rangle - |1s\rangle_{r_2}|\uparrow\rangle|1s\rangle_{r_1}|\uparrow\rangle = 0^2$ ← Pauli!

Consequences of Indistinguishability & Solids

15 Feb

Review of Last Time

- If the particles are ~~dt~~ indistinguishable, then
  - → All observables must be symmetric under interchange (total spin)
  - → The wave fcn must be symmetric as well

Σ

- $\exists$ 2 ways to do this

$$|\psi\rangle = |\psi_1 \psi_2\rangle \pm |\psi_2 \psi_1\rangle$$

The + is for bosons (Spin = $n\hbar$)
The − is for fermions (spin = $\frac{2n+1}{2}\hbar$)

Slide Ex Three for them to do (ans on prior page)

What if I have more than 2 particles? Generalize!

If I have more than 2, I need Ψ to be (anti)symmetric under the interchange of any two particles.

The formal way to do this is through the introduction of the exchange operator $\hat{P}$

$$P|\psi_1 \psi_2\rangle = |\psi_2 \psi_1\rangle$$

Ex What is the eigenvalue of $\hat{P}^2$ (there is only 1 ψ)
- 1.5 min on your own
- Who wants to converse w/ neighbor?
  → Do iff wanted
- 1 as you return to where you started.

argument ≠ proof

If $\hat{P}^2|\psi\rangle = 1\cdot|\psi\rangle$ then a sloppy argument of "square root both sides" ~~in~~ suggests

$\hat{P}$ has eigenvalues ± 1

(you can prove ☺). This is correct.

$$\hat{P}|\psi\rangle = \pm 1|\psi\rangle$$

- + for bosons
- − for fermions

Thus, formally, for a system of n identical particles

$$P|1,2,3,\ldots,i,\ldots,j,\ldots n\rangle = \pm|1,2,3,\ldots,j,\ldots,i,\ldots n\rangle$$

w/ again + for bosons & − for fermions

A bit more about $\hat{P}$

Ex Slide ~~If,~~ For a properly constructed Hamiltonian of identical particles, does $\hat{P}$ commute w/ $\hat{H}$?

A) Y
B) N

A proper H must be (anti) symmetric under interchange!

Slide Will the "(anti)symmetric-ness" ever change with time?

Hint: Think about $\langle \hat{P} \rangle$. Thinking way back to the start of QM I, how do expectation values evolve w/ time?

A) Y
B) N

$$\frac{d\langle \hat{O} \rangle}{dt} = [\hat{O}, \hat{H}]$$

here $[\hat{P}, \hat{H}] = 0$ so it will remain (anti)symmetric forever!

HW Ref. Your 2nd HW problem (Griffiths 5.8) will explore how to do this in the 3 Particle case.

Exchange Forces (Griffiths 5.1.2)

Quiz Ref I am NOT going to go into too much depth on the general 1-D derivation that Griffiths does here. I think his derivation is very reasonable & clear. At the very least read (& ideally work through) his derivation on 203-205. This will be your quiz for next week!

Long-story-short:

- For two particles their separation is

$$\langle (x_1 - x_2)^2 \rangle$$

HW1 Ref

- As you saw in Q3 of HW1, for distinguishable particles

Keep

$$\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\langle x_1 x_2 \rangle$$

HW2 Ref

- You will prove in Q1 of HW2 (& see also in the ~~der~~ more general derivation in 5.1.2 that for indistinguishable particles in states a & b that:

$$\langle (x_1 - x_2)^2 \rangle = \langle x \rangle_a + \langle x \rangle_b - 2\langle x_a \rangle \langle x \rangle_b \mp 2|\langle x \rangle_{ab}|^2$$

where $\langle x \rangle_{ab} = \int \psi_a^*(x)\, x\, \psi_b(x)\, dx$

& top sign is for bosons & bottom fermions

The difference here is the $\mp 2|\langle x \rangle_{ab}|^2$ term
- Bosons (− sign) are closer than distinguishable particles
- Fermions (+ sign) are further apart.

The effect of exchange tends to
- Bring bosons together (pseudo attractive force)
- Drive fermions apart (pseudo repulsive force)

While NOT actually forces, called ~~[illegible]~~ exchange forces b/c what this effect does!