Physics 564, Spring 2025
Midterm Prep Summary

1 Topics

1. Basics of group theory:
   (a) Definition of a group.
   (b) Dimensionality.
   (c) Representations.
   (d) Continuous Groups $\rightarrow$ Lie Groups.
2. Applications of Group theory:
   (a) Parity.
   (b) Space translation.
   (c) Time translation.
   (d) Rotations.
3. Addition of angular momentum (*Griffiths* 4.4 - though this requies a strong grasp of the general theory of quantum mechanical angular momentum discussed in section 4.3).

2 Practice Problems

1. Consider the tritium nucleon p^+, n^0, n^0. Assuming the ground state is $L = 0$, what is the eigenvalue S^2? What are the possible eignenvalues of S_z?
2. Consider a 1-D harmonic oscillator with Hamiltonian
   $$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2 = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$
   The eigenstates, of course, are $|n\rangle$, $n \in 0, 1, 2, 3, ...$ Consider the following operator called the Glauber discplacement (R.J. Glauber won the 2005 Nobel Prize in physics.)
   $$D(\alpha) = \exp\left(\alpha\hat{a}^\dagger - \alpha^*\hat{a}\right),$$

where α is a complex number.

(a) Show that $D(\alpha)$ is unitary.
Hint: You may need the operator identity $e^{(A+B)} = e^A e^B e^{-\frac{1}{2}[A,B]}$.

(b) Find $\langle 0|D(\alpha)|0\rangle$.

(c) Find $\langle 0|D^\dagger(\alpha) x D(\alpha)|0\rangle$.

3. Griffiths 6.7

4. Griffiths 6.12

5. Griffiths 6.17

6. Griffiths 6.18

7. Griffiths 6.27

8. Griffiths 6.36

9. Griffiths 6.37