(Griffiths 5.19) The ground state of dysprosium is given as $^5I_8$.

What are the total spin, total orbital, and grand total angular momentum quantum numbers?

What is a likely electron configuration for dysprosium?

$$^5I_8 \Rightarrow 5 = 2S+1 \rightarrow S = 2$$

$$I \rightarrow L = 7$$

$$8 \rightarrow J = 8$$

For the electron configuration, we can read from the periodic table

$$(1s)^2(2s)^2(2p)^6(3s)^2(3p)^6(4s)^2(3d)^{10}$$
$$(4p)^6(5s)^2(4d)^{10}(5p)^6(6s)^2(4f)^{10}$$

or more compactly

$$(Xe)(6s)^2(4f)^{10}$$

You can see a discussion on 4s having lower energy than 3d in the 3rd paragraph of page 213 I asked you to read.

Homework 7

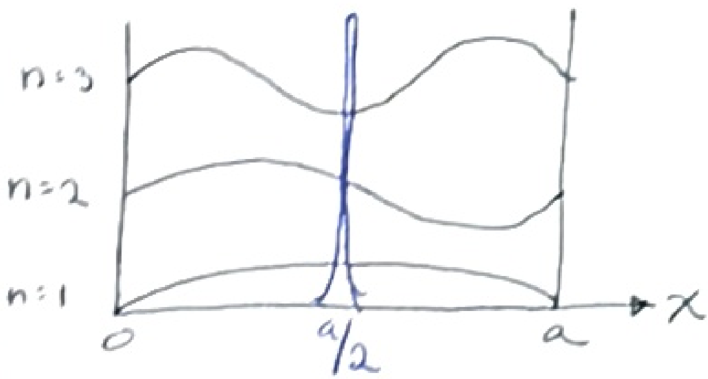
1A

1. (Griffiths 7.1) Suppose we put a delta-fcn bump at the center of the square well:

$$H' = \alpha\,\delta(x - a/2)$$

a) Find the first order perturbed energies. Why are energies not perturbed for even n?

The second part is straightforward: all even-n are zero at the center of the well so are unaffected by the $\delta(x - a/2)$.



Now to find the energy corrections:

$$E_n^{(1)} = \langle n | H' | n \rangle$$

$$= \int dx \, \sin\left(\frac{n\pi x}{a}\right) \alpha\,\delta(x - a/2) \sin\left(\frac{n\pi x}{a}\right) \cdot \left(\sqrt{\frac{2}{a}}\right)^2$$

$$= \frac{2}{a}\alpha \sin^2\left(\frac{\pi n}{\cancel{a}} \cdot \frac{\cancel{a}}{2}\right) = \frac{2\alpha}{a}\sin^2\left(\frac{n\pi}{2}\right)$$

HW #7

b) Find the first 3 non-zero terms in $\psi_1^{(1)}$.

$$\psi_n^{(1)} = \sum_{m \neq n} \frac{\langle \psi_m^{(0)} | H' | \psi_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} | \psi_m^{(0)} \rangle$$

with $n=1$

$$\psi_1^{(1)} = \sum_{m \neq 1} \frac{\langle \psi_m^{(0)} | H' | \psi_1^{(0)} \rangle}{E_1^{(0)} - E_m^{(0)}} | \psi_m^{(0)} \rangle$$

Let's look at the numerator:

$$\langle \psi_m^{(0)} | H' | \psi_n^{(0)} \rangle = \int dx \left(\sqrt{\frac{2}{a}}\right)^2 \sin\left(\frac{m\pi x}{a}\right) \alpha \delta(x - a/2) \sin\left(\frac{\pi x}{a}\right)$$

$$= \frac{2\alpha}{a} \sin\left(\frac{m\pi}{2}\right) \sin\left(\frac{\pi}{2}\right)$$

$\sin(\pi/2) = 1$ so

$$\langle \psi_m^{(0)} | H' | \psi_n^{(0)} \rangle = \frac{2\alpha}{a} \sin\left(\frac{m\pi}{2}\right)$$

which is non-zero for odd n. Since $m=1$ is excluded, we have $m \in \{3, 5, 7\}$

Meanwhile, in the denominator, we have

HW #7

$$E_n^{(0)} - E_m^{(0)} = \frac{\pi^2\hbar^2 n^2}{2ma^2} - \frac{\pi^2\hbar^2 m^2}{2ma^2}, \text{ but } n=1 \text{ so}$$

$$= \frac{\pi^2\hbar^2}{2ma^2}\left(1 - m^2\right)$$

and

$$\frac{1}{E_n^{(0)} - E_m^{(0)}} = \frac{2ma^2}{\pi^2\hbar^2\left(1-m^2\right)}$$

Putting it all together, and recognizing that $\sin(3\pi/2) = -1$, $\sin(5\pi/2) = +1$, $\sin(7\pi/2) = -1$, we have

$$\psi_1^{(1)} = \frac{2\alpha}{\cancel{\alpha}} \cdot \frac{2ma^2}{\pi^2\hbar^2}\left[\frac{-1}{1-9}\psi_3^{(0)} + \frac{1}{1-25}\psi_5^{(0)} - \frac{1}{1-49}\psi_7^{(0)} + \ldots\right]$$

$$\psi_1^{(1)} = \frac{4m\alpha a}{\pi^2\hbar^2}\sqrt{\frac{2}{a}}\left[\frac{1}{8}\sin\left(\frac{3\pi x}{a}\right) - \frac{1}{24}\sin\left(\frac{5\pi x}{a}\right) + \frac{1}{48}\sin\left(\frac{7\pi x}{a}\right) + \ldots\right]$$

$$= \frac{m\alpha}{\pi^2\hbar^2}\sqrt{\frac{a}{2}}\left[\sin\left(\frac{3\pi x}{a}\right) - \frac{1}{3}\sin\left(\frac{5\pi x}{a}\right) + \frac{1}{6}\sin\left(\frac{7\pi x}{a}\right) + \ldots\right]$$

HW #7 2A

2. (Griffiths 7.2) For the 1-D QMHO,

$$H = \frac{p^2}{2m} + \frac{k}{2}x^2 \text{ with } E_n = (n+\tfrac{1}{2})\hbar\omega$$

with $\omega = \sqrt{\frac{k}{m}}$

Now perturb by $k \to (1+\varepsilon)k$.

a) Find the exact solution and expand to 2^{nd} order in ε.

As $k \to (1+\varepsilon)k$, $\omega \to \sqrt{\frac{(1+\varepsilon)k}{m}} = \omega'$

so the energies are $E_n = (n+\frac{1}{2})\hbar\omega'$, or, in terms of k & ε:

$$E_n = \hbar(n+\tfrac{1}{2})\sqrt{\frac{k}{m}(1+\varepsilon)} = \hbar\omega(n+\tfrac{1}{2})\sqrt{1+\varepsilon}$$

$$(1+\varepsilon)^n = 1 + n\varepsilon + \frac{1}{2}n(n-1)\varepsilon^2 + \dots$$

$$E_n = (n+\tfrac{1}{2})\hbar\omega\left[1 + \frac{1}{2}\varepsilon + \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\varepsilon^2 + \dots\right]$$

$$E_n = (n+\tfrac{1}{2})\hbar\omega\left[1 + \frac{1}{2}\varepsilon - \frac{1}{8}\varepsilon^2 + \dots\right]$$

HW #7 2B

b) Now use perturbation theory (without integrals!) to find $E_n^{(1)}$ and see that it agrees with part a.

$$E_n^{(1)} = \langle n | H' | n \rangle$$

What is H'?

$$H = \frac{p^2}{2m} + \frac{k(1+\varepsilon)}{2} x^2 .$$ Now expand:

$$H = \underbrace{\frac{p^2}{2m} + \frac{k}{2} x^2}_{H_0} + \underbrace{\frac{k\varepsilon}{2} x^2}_{H'}$$

$$\therefore E_n^{(1)} = \langle n | H' | n \rangle$$

$$= \left\langle n \middle| \frac{k\varepsilon}{2} x^2 \middle| n \right\rangle$$

$\exists$ 2 ways to do this:

1) The virial theorem as x^n with $n=2$ $\quad \langle T \rangle = \frac{n}{2} \langle V \rangle \xrightarrow[\text{QMHO}]{\text{for}} \langle T \rangle = \langle V \rangle$
2) Operators

The virial theorem method is most straightforward

$$E_n^{(1)} = \varepsilon \langle n | V | n \rangle = \varepsilon \langle V \rangle$$

HW #7

Since $\langle V \rangle = \langle T \rangle$ and $\langle V \rangle + \langle T \rangle = E_n$, $\langle V \rangle = \frac{1}{2} E_n$. Thus

$$E_n^{(1)} = \frac{\varepsilon}{2} E_n = \frac{\varepsilon}{2} (n + 1/2) \hbar \omega$$

a perfect match for the linear term highlighted at the bottom of pg 2A.

The other option is to write $x = \sqrt{\frac{\hbar}{2m\omega}} (a + a^+)$ and proceed that way.

Problem 3 (Griffiths 7.4)

Start by stating global assumptions for this notebook: that both energies are real and positive while $0 < \lambda < 1$.

In[1]:=
```
$Assumptions = Ea ∈ Reals && Ea > 0 && Eb ∈ Reals && Eb > 0 && λ ∈ Reals && λ > 0 && λ < 1
```
Out[1]= $Ea \in \mathbb{R}$ && $Ea > 0$ && $Eb \in \mathbb{R}$ && $Eb > 0$ && $\lambda \in \mathbb{R}$ && $\lambda > 0$ && $\lambda < 1$

Apply perturbation theory to the most general two-state system with unperturbed energies:

In[2]:=
```
MatrixForm[H0 = {{Ea, 0}, {0, Eb}}]
```
Out[2]//MatrixForm=

$$\begin{pmatrix} Ea & 0 \\ 0 & Eb \end{pmatrix}$$

---

and perturbation

In[3]:=
```
MatrixForm[H1 = λ {{Vaa, Vab}, {Vab, Vbb}}]
```
Out[3]//MatrixForm=

$$\begin{pmatrix} Vaa\,\lambda & Vab\,\lambda \\ Vab\,\lambda & Vbb\,\lambda \end{pmatrix}$$

(Note, I am using Vab = Vba. What is really true, however, Vba* = Vab. However, this is simpler for Mathematica)

In[4]:=
```
MatrixForm[H1]
```
Out[4]//MatrixForm=

$$\begin{pmatrix} Vaa\,\lambda & Vab\,\lambda \\ Vab\,\lambda & Vbb\,\lambda \end{pmatrix}$$

a) Find the exact energies

In[5]:=
```
MatrixForm[H = H0 + H1]
```
Out[5]//MatrixForm=

$$\begin{pmatrix} Ea + Vaa\,\lambda & Vab\,\lambda \\ Vab\,\lambda & Eb + Vbb\,\lambda \end{pmatrix}$$

In[6]:= **Emp = Eigenvalues[H]**

Out[6]= $$\left\{\frac{1}{2}\left(\text{Ea}+\text{Eb}+\text{Vaa}\,\lambda+\text{Vbb}\,\lambda-\sqrt{\left((-\text{Ea}-\text{Eb}-\text{Vaa}\,\lambda-\text{Vbb}\,\lambda)^2-4\left(\text{Ea}\,\text{Eb}+\text{Eb}\,\text{Vaa}\,\lambda+\text{Ea}\,\text{Vbb}\,\lambda-\text{Vab}^2\lambda^2+\text{Vaa}\,\text{Vbb}\,\lambda^2\right)\right)}\right),\ \frac{1}{2}\left(\text{Ea}+\text{Eb}+\text{Vaa}\,\lambda+\text{Vbb}\,\lambda+\sqrt{\left((-\text{Ea}-\text{Eb}-\text{Vaa}\,\lambda-\text{Vbb}\,\lambda)^2-4\left(\text{Ea}\,\text{Eb}+\text{Eb}\,\text{Vaa}\,\lambda+\text{Ea}\,\text{Vbb}\,\lambda-\text{Vab}^2\lambda^2+\text{Vaa}\,\text{Vbb}\,\lambda^2\right)\right)}\right)\right\}$$

Which simplifies into E- (Em) and E+ (Ep) via

In[7]:= **Simplify[(a + b)^2 - 4 a * b]**

Out[7]= $(a-b)^2$

In[8]:= **Em = 1 / 2 * ((Ea + Eb + Vaa * λ + Vbb * λ) - Sqrt[(Ea + λ * Vaa - Eb - λ * Vbb)^2 + 4 * λ^2 * Vab^2])**

Out[8]= $$\frac{1}{2}\left(\text{Ea}+\text{Eb}+\text{Vaa}\,\lambda+\text{Vbb}\,\lambda-\sqrt{4\,\text{Vab}^2\lambda^2+(\text{Ea}-\text{Eb}+\text{Vaa}\,\lambda-\text{Vbb}\,\lambda)^2}\right)$$

In[9]:= **Ep = 1 / 2 * ((Ea + Eb + Vaa * λ + Vbb * λ) + Sqrt[(Ea + λ * Vaa - Eb - λ * Vbb)^2 + 4 * λ^2 * Vab^2])**

Out[9]= $$\frac{1}{2}\left(\text{Ea}+\text{Eb}+\text{Vaa}\,\lambda+\text{Vbb}\,\lambda+\sqrt{4\,\text{Vab}^2\lambda^2+(\text{Ea}-\text{Eb}+\text{Vaa}\,\lambda-\text{Vbb}\,\lambda)^2}\right)$$

b) Expand your results from (a) to second order in λ. Show that the results are the same as what we obtained in perturbation theory.

In[10]:= **Series[Em, {λ, 0, 2}]**

Out[10]= $$\frac{1}{2}(\text{Ea}+\text{Eb}-\text{Abs}[\text{Ea}-\text{Eb}])+\frac{1}{2}\left(\text{Vaa}+\text{Vbb}-\frac{(\text{Vaa}-\text{Vbb})\,\text{Abs}[\text{Ea}-\text{Eb}]}{\text{Ea}-\text{Eb}}\right)\lambda-\frac{\text{Vab}^2\,\text{Abs}[\text{Ea}-\text{Eb}]\,\lambda^2}{(\text{Ea}-\text{Eb})^2}+O[\lambda]^3$$

This, of course, simplifies to (Ea-Eb)^2 = (Eb - Ea)^2

In[11]:= **Em = 1 / 2 (2 * Eb - Ea + Ea) + 1 / 2 (Vaa + Vbb - Vaa + Vbb) * λ + (Vab^2 * λ^2) / (Eb - Ea)**

Out[11]= $$\text{Eb}+\text{Vbb}\,\lambda+\frac{\text{Vab}^2\lambda^2}{-\text{Ea}+\text{Eb}}$$

Similarly for Ep

In[12]:= **Series[Ep, {λ, 0, 2}]**

Out[12]= $$\frac{1}{2}(\text{Ea}+\text{Eb}+\text{Abs}[\text{Ea}-\text{Eb}])+\frac{1}{2}\left(\text{Vaa}+\text{Vbb}+\frac{(\text{Vaa}-\text{Vbb})\,\text{Abs}[\text{Ea}-\text{Eb}]}{\text{Ea}-\text{Eb}}\right)\lambda+\frac{\text{Vab}^2\,\text{Abs}[\text{Ea}-\text{Eb}]\,\lambda^2}{(\text{Ea}-\text{Eb})^2}+O[\lambda]^3$$

Simplifies to

In[13]:=
```
Ep = 1/2 (2 * Ea - Eb + Eb) + 1/2 (Vaa + Vaa - Vbb + Vbb) λ + (Vab^2 * λ^2) / (Eb - Ea)
```

Out[13]=

$$\mathrm{Ea} + \mathrm{Vaa}\,\lambda + \frac{\mathrm{Vab}^2\,\lambda^2}{-\mathrm{Ea} + \mathrm{Eb}}$$

Both of which match our results from perturbation theory!

c) Setting Vaa = Vbb = 0, show that the series only converges if | Vab / (Eb - Ea) | < 1/2

The series only converge if

In[14]:=

```
4 Vab^2 / (Eb - Ea)^2 < 1
```

Out[14]=

$$\frac{4\,\mathrm{Vab}^2}{(-\mathrm{Ea} + \mathrm{Eb})^2} < 1$$

or equivalently

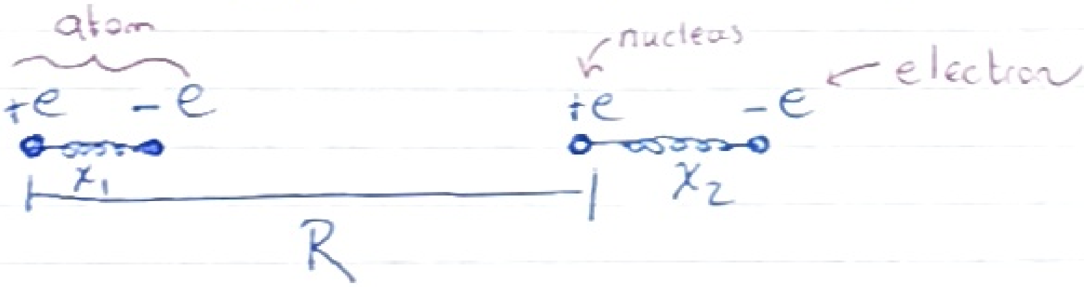
In[15]:=
```
| Vab / (Eb - Ea) | < 1/2
```

HW#7

4 (Griffiths 7.39) Van der Waals

$$H^0 = \frac{p_1^2}{2m} + \frac{1}{2}kx_1^2 + \frac{p_2^2}{2m} + \frac{1}{2}kx_2^2$$

$$H' = \frac{1}{4\pi\varepsilon_0}\left(\frac{e^2}{R} - \frac{e^2}{(R-x_1)} - \frac{e^2}{(R+x_2)} + \frac{e^2}{(R-x_1+x_2)}\right)$$



a) Explain the perturbation.

The first term is the nuclear repulsion: $+e^2/R$

The second term is the attraction between the electron on the left and the nucleus on the right: $-e^2/(R-x_1)$

The third term is the attraction between the electron on the right and the left nucleus: $-e^2/(R+x_2)$

The final term is the repulsion of the two electrons: $+\dfrac{e^2}{R-x_1+x_2}$

HW #7

Also, assuming $|x_1| \ll R$ & $|x_2| \ll R$, show

$$H' \simeq \frac{-e^2 x_1 x_2}{2\pi\varepsilon_0 R^3}$$

$\frac{1}{R-x}$ appears in all terms but the first in some form. We want $(1+\varepsilon)^n$

$$\frac{1}{R-x} = \frac{1}{R}\left(1-\frac{x}{R}\right)^{-1}$$

$$\sim \frac{1}{R}\left(1-\frac{x}{R} + \frac{1}{2}(-1)(-1-1)\left(\frac{x}{R}\right)^2 + \cdots\right)$$

$$\sim \frac{1}{R}\left(1-\frac{x}{R} + \left(\frac{x}{R}\right)^2 + \cdots\right)$$

$$H' \approx \frac{e^2}{4\pi\varepsilon_0 R}\left[\cancel{1} - \left(\cancel{1} + \frac{x_1}{R} + \frac{x_1^2}{R^2}\right) - \left(\cancel{1} - \frac{x_2}{R} + \frac{x_2^2}{R^2}\right) + \left(\cancel{1} + \frac{(x_1-x_2)}{R} + \frac{(x_1-x_2)^2}{R^2}\right)\right]$$

$$\approx \frac{e^2}{4\pi\varepsilon_0 R}\left[\cancel{\frac{-x_1}{R}} + \cancel{\frac{x_2}{R}} + \cancel{\frac{(x_1-x_2)}{R}} - \cancel{\frac{x_1^2}{R}} - \cancel{\frac{x_2^2}{R}} + \cancel{\frac{x_1^2}{R}} + \cancel{\frac{x_2^2}{R}} - \frac{2x_1x_2}{R}\right]$$

$$\approx -\frac{e^2 x_1 x_2}{2\pi\varepsilon_0 R^2}$$ ο.ε.δ.

P564

HW #7 4C

b) Show that H_0 plus the simplified H' separates into 2 QMHO's:

$$H = \left[\frac{p_+^2}{2m} + \frac{1}{2}\left(k - \frac{e^2}{2\pi\varepsilon_0 R^3}\right)x_+^2\right] + \left[\frac{p_-^2}{2m} + \frac{1}{2}\left(k + \frac{e^2}{2\pi\varepsilon_0 R^3}\right)x_-^2\right]$$

With $x_\pm = \frac{1}{\sqrt{2}}(x_1 \pm x_2)$ $\quad p_\pm = \frac{1}{\sqrt{2}}(p_1 \pm p_2)$

The easiest way to do this is to work backwards and expand the result

$$p_+^2 = (p_1^2 + p_2^2 + 2p_1p_2)\,{}^1\!/_2$$

(since 1 & 2 are independent particles, p_1 & p_2 commute!)

Similarly:

$$p_-^2 = \tfrac{1}{2}(p_1^2 + p_2^2 - 2p_1p_2)$$

$$x_+^2 = \tfrac{1}{2}(x_1^2 + x_2^2 + 2x_1x_2)$$

$$x_- = \tfrac{1}{2}(x_1^2 + x_2^2 - 2x_1x_2)$$

$$p_+^2 + p_-^2 = p_1^2 + p_2^2$$

$$x_+^2 + x_-^2 = x_1^2 - x_2^2 \qquad x_-^2 - x_+^2 = -2x_1x_2 \quad \text{O.E.D.}$$

HW #7 4D

c) Assuming $k >> \left(\frac{e^2}{2\pi\varepsilon_0 R^3}\right)$ show that

$$\Delta V = E - E_0 \cong -\frac{\hbar}{8m^2\omega_0^3}\left(\frac{e^2}{2\pi\varepsilon_0}\right)^2 \frac{1}{R^6}$$

with $\omega_0 = \sqrt{k/m}$ & $E_0 = (n_1 + n_2 + 1)\hbar\omega = \hbar\omega$

The energies, in this case, are

$$E_0 = (n_+ + \tfrac{1}{2})\omega_+\hbar + (n_- + \tfrac{1}{2})\hbar\omega_- = \tfrac{1}{2}\hbar(\omega_+ + \omega_-)$$
for $n_+ = n_- = 0$.

$$\omega_\pm = \sqrt{\frac{k \mp (e^2/2\pi\varepsilon_0 R^3)}{m}}$$

$$\omega_\pm = \sqrt{\frac{k}{m}}\left(1 \mp \frac{e^2}{2\pi\varepsilon_0 R^3 k}\right)^{1/2}$$

$$\omega_\pm \approx \sqrt{\frac{k}{m}}\left[1 \mp \frac{1}{2}\left(\frac{e^2}{2\pi\varepsilon_0 R^3 k}\right) + \frac{1}{2}\left(\frac{+1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{\mp e^2}{2\pi\varepsilon_0 R^3 k}\right)^2 + \cdots\right]$$

$$\approx \sqrt{\frac{k}{m}}\left[1 \mp \frac{1}{2}\left(\frac{e^2}{2\pi\varepsilon_0 R^3 k}\right) - \frac{1}{8}\left(\frac{e^2}{2\pi\varepsilon_0 R^3 k}\right)^2 + \cdots\right]$$

$$\Delta V = \tfrac{1}{2}\hbar(\omega_+ + \omega_-) - \hbar\omega_0$$

P5G4

HW #7 4E

$$\Delta V \approx \frac{1}{2}\hbar\omega_0 \left\{ \left[1 - \cancel{\frac{1}{2}\left(\frac{e^2}{2\pi\varepsilon_0 R^3 k}\right)} - \frac{1}{8}\left(\frac{e^2}{2\pi\varepsilon_0 R^3 k}\right)^2 \right] + \left[1 + \cancel{\frac{1}{2}\left(\frac{e^2}{2\pi\varepsilon_0 R^3 k}\right)} - \frac{1}{8}\left(\frac{e^2}{2\pi\varepsilon_0 R^3 k}\right)^2 \right] \right\} - \hbar\omega_0$$

$$\Delta V \approx \frac{1}{\cancel{2}}\hbar\omega_0 \left\{ \cancel{2} - \frac{\cancel{2}}{8}\left(\frac{e^2}{2\pi\varepsilon_0 R^3 k}\right)^2 \right\} - \cancel{\hbar\omega_0}$$

$$\Delta V \approx -\frac{\hbar\omega_0}{8}\frac{e^2}{(2\pi\varepsilon_0 R^3 k)^2} \qquad k = m\omega_0^2$$

$$\approx -\frac{1}{8}\frac{\hbar}{m^2\omega_0^3}\left(\frac{e^2}{2\pi\varepsilon_0}\right)^2 \frac{1}{R^6}$$

d) Now repeat using 2^{nd} order P.T.

At first order, things go to zero

$E_0^{(1)} = \langle 00 | H' | 00 \rangle$ using $|x_1 x_2\rangle$ notation

$$= -\frac{e^2}{2\pi\varepsilon_0 R^3} \langle 00 | x_1 x_2 | 00 \rangle$$

Since $x_1 = \sqrt{\frac{\hbar}{2m\omega}}(a_1 + a_1^+)$, we see that the result is zero: a_1 by lowering $|0\rangle \to 0$ and a_1^+ by orthogonality. The same will be true for x_2

HW#7 4F

At second order

$$E_0^{(2)} = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} \frac{|\langle 00|H'|n_1n_2\rangle|^2}{E_{00}-E_{n_1n_2}}$$

$$= \left(\frac{e^2}{2\pi\varepsilon_0 R^3}\right)^2 \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} \frac{1}{E_{00}-E_{n_1n_2}} \left|\langle 00|x_1x_2|n_1n_2\rangle\right|^2$$

Again, recalling $x_i = \sqrt{\frac{\hbar}{2m\omega}}(a_i + a_i^+)$
the only non-zero term is $|11\rangle$ as all others go to zero by orthogonality.

$$E_{11} = \hbar\omega_0(1+1+1) = 3\hbar\omega_0$$

$$E_{00} = \hbar\omega_0(0+0+1) = \hbar\omega_0$$

$$E_{00} - E_{11} = -2\hbar\omega_0$$

$$\left|\langle 00|x_1x_2|11\rangle\right|^2 = \left|\sqrt{\frac{\hbar}{2m\omega}}\sqrt{\frac{\hbar}{2m\omega}}\sqrt{1}\sqrt{1}\right|^2$$

$$= \left(\frac{\hbar}{2m\omega}\right)^2$$

$$\therefore E_0^{(2)} = \left(\frac{e^2}{2\pi\varepsilon_0}\right)^2\left(\frac{-1}{2\hbar\omega_0}\right)\left(\frac{\hbar}{2m\omega}\right)^2\frac{1}{R^6}$$

$$E_0^{(2)} = \frac{-\hbar}{8m^2\omega_0^3}\left(\frac{e^2}{2\pi\varepsilon_0}\right)^2\frac{1}{R^6} \quad \text{o.ε.δ.}$$