Physics 564, Spring 2025
Homework assignment 4
Due Tuesday, 11 March

Remember that your homework solutions must contain a description of what you are doing, and *not* just a list of equations.

1. *This was problem 4 on last week's homework. While not required last week, many of you did it. If you did, you can ignore it this time. If you did not do it last week, do it this week!*

   The 4-rep of $\mathfrak{su}(2)$: In class, we explored the 2-rep and the 3-rep. I would like you to create the 4-rep:

   (a) What are the generators T_4^j? How many generators must there be? In what basis did you write them?

   (b) Determine the Casimir operator $C(4)$ for $\mathfrak{su}(2)$.

2. Complete, and turn in, your worksheet on angular momentum review that we began in class!

3. **Crystals as application of the translation operator:** Some electrons in a solid are free to move about the crystal (the so called "sea" electrons). Let's explore a one-dimensional crystal as an example. For electrons in such a crystal, the potential energy is periodic, i.e.

   $$V(x+a) = V(x),$$

   where a is the distance between one lattice site and the next. Since the potential is periodic, the mod-square of the wave function must be as well

   $$|\psi(x+a)|^2 = |\psi(x)|^2.$$

   Of course, the edges of the crystal will mess up this pattern. However, for a real material there are order Avogadro's number ($\sim 10^{23}$) of lattice sites. It seems, therefore, unreasonable that an electron in the middle of the crystal would be impacted by the edges. The solution is to impose a *periodic boundry condition*, i.e. bend the crystal into a circle. In this case, if there are N lattice sites around the crystal and I go all the way around, I must return to where I started:

$$\psi(x + Na) = \psi(x).$$

(a) Use the translation operator for the last homework to write the wave function $\psi(x+a)$ in terms of $\psi(x)$ and a crystal momentum q, which is the *group velocity* of the electron wave as it moves through the crystal.

Note: There is a sign difference with last week where we looked at translating $\psi(x)$ to $\psi(x-a)$!

(b) The periodic boundry condition described above imposes a quantization condition on q. Show why this is true and find that condition.

Also, do not forget that you can also turn in your next metacognitive exercise that same day.