Physics 564, Spring 2025
Homework assignment 3
Due Tuesday, 4 March

# Remember that your homework solutions must contain a description of what you are doing, and *not* just a list of equations.

1. In class, we discussed at length the time-evolution operator

$$\mathcal{T}(t) = \exp\left[-\frac{it}{\hbar}H\right]$$

Here, I want you to explore an operator that translates in space:

$$\mathcal{P}(a)\psi(x) = \psi(x-a)$$

Following the process, we did in class, determine the form of $\mathcal{P}$.

2. Read section 6.6 in *Griffiths* and do problem 6.18

3. **Orthogonality of generators:** If the generators of a Lie algebra are thought to span the space of the plane tangent to the manifold of the group, it is probably easiest if they are orthogonal (we usually work in orthogonal coordinate systems like $\hat{x}, \hat{y}, \hat{z}$ after all!). What does it mean for matricies to be orthogonal? It means that the trace of any pair of generators follows the rule,

$$\mathrm{tr}[T_r^a T_r^b] = C(r)\delta_{ab},$$

where $T_r^a$ and $T_r^b$ are generators of the algebra in some representation $r$ and $C(r)$ is a constant, called the *quadratic Casimir operator*[1] which is representation-dependent.

You should know from class that the Paui matricies are the 2-rep $\mathfrak{su}(2)$[2]

[1]These Casimir operators appear very often in non-Abelian gauge theories.

[2]By the way to make this text in LaTeX, you would write $\mathrfrak{su}(2)$. Which requires \usepackage{amsfonts}.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Show that the 2-rep $\mathfrak{su}(2)$ meet the orthogonality condition above. Determine $C(2)$ and show it is constant for all pairs.

4. **The 4-rep of $\mathfrak{su}(2)$:** In class, we explored the 2-rep and the 3-rep. I would like you to create the 4-rep:
   (a) What are the generators $T_4^j$? How many generators must there be? In what basis did you write them?
   (b) Determine the Casimir operator $C(4)$ for $\mathfrak{su}(2)$.

**Also, do not forget that you can also turn in your next metacognitive exercise that same day.**