Physics 564, Spring 2024
Homework assignment 1
Due TBD

Remember that your homework solutions must contain a description of what you are doing, and *not* just a list of equations.

1. Finish problem 8 from our review worksheet:

   Using $\langle x^2 \rangle$ and $\langle p^2 \rangle$, prove the virial theorem in the case of the harmonic oscillator:

   $$\langle V \rangle = \langle T \rangle = \frac{\langle H \rangle}{2}$$

2. Similar to what we did in class for momentum, prove that a symmetry in time results in conservation of energy:

   Start with a Lagrangian $L(x_i, \dot{x}_i, t)$ in a closed system (so $\partial L/\partial t = 0$). Next, consider the total derivative of the Lagrangian w.r.t. time dL/dt.

   You *will* need to assume that the potential energy U is independent of time and generalized velocity, i.e.

   $$U(x_i) \text{ and } \frac{\partial U}{\partial \dot{x}_i} = \frac{\partial U}{\partial t} = 0.$$

3. Which of the following are groups?

   (a) All real numbers (group multiplication = ordinary multiplication).

   (b) All real numbers (group multiplication = addition).

   (c) All complex numbers *except* zero (group multiplication = ordinary multiplication).

   (d) All real numbers (group multiplication of a and $b = \sqrt{ab}$).

   (e) All complex numbers (group multiplication of a and $b = \sqrt{ab}$).

4. Work out the symmetry group of a square. How many elements does it have? Construct the multiplicaton table, and determine whether or not the group is Abelian.

5. Of particular importance in physics are the *unitary* and *orthogonal* groups. Unitary groups can be represented by unitary matricies: $U^\dagger = U^{-1} \rightarrow U^\dagger U = 1$. Orthogonal groups are similar except for all the elements are real implying that $O^T = O^{-1}$.

   Show that the set of all unitary $n \times n$ matricies constitutes a group.

   Hint: To show closure, for example, you must show that the product of two unitary matricies is itself unitary.

Don't forget, you can also complete your first metacognitive exercise on Gradescope (linked from Canvas)