Quantum I Review

This is a series of questions to help you review QMI. Work with your peers and feel free to use your book!

Notes for these problems

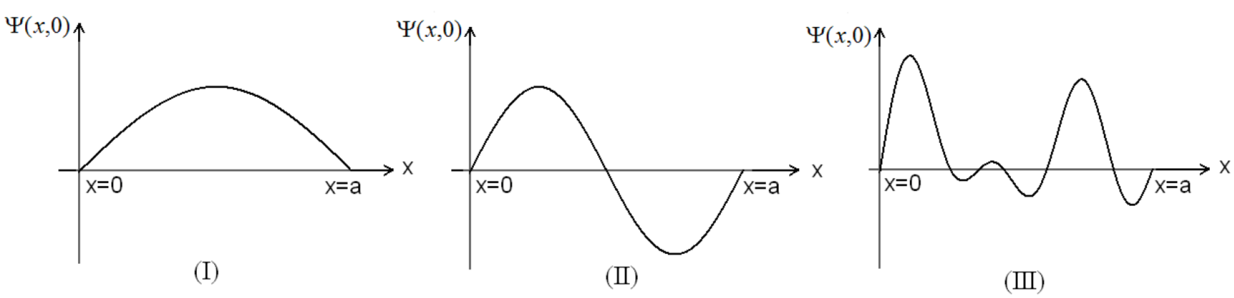
- $\hat{x}$, $\hat{p}$, and $\hat{H}$ correspond to the position, momentum and Hamiltonian operators respectively for a given quantum system.
- A physical observable is "well-defined" when it has a definite value for a given wave function.
- "1-D" is an abbreviation for one-dimensional. All questions refer to systems with one spatial dimension.
- $\psi(x)$ is a time-independent wave function. $\Psi(x,t)$ is a function at time t.

1-D Infinite Square Well

A particle sits inside a 1-D infinite square well of width a:

$$V(x) = \begin{cases} 0, & 0 \le x \le a \\ \infty, & \text{otherwise} \end{cases}$$

1. Choose all the possible wave functions $\Psi(x, t = 0)$ for this system (all graphs are continuous and differentiable for $0 \le x \le a$.



2. Suppose at $t = 0$ the particle is in the first excited state. Which of the following expectation values depend on time $\langle \hat{x} \rangle$, $\langle \hat{p} \rangle$, $\langle \hat{H} \rangle$?

3. If the particle is in the $n = 5$ state, what is the probability density as a function of time?

4. The wave function at some time is

$$\Psi(x,t) = \sqrt{\frac{2}{7}}\psi_1(x) + i\sqrt{\frac{5}{7}}\psi_2(x)$$

   a. What are the possible values for the energy? Just a list of E_n is sufficient.

   b. What is the expectation value of the energy as a function of E_n?

   c. If a measurement is made and the resulting energy is $\frac{4\pi^2\hbar^2}{2ma^2}$ what is the spatial wavefunction immediately after the measurement?

5. A particle is in the state

$$\Psi(x,t) = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$$

   a. which of the following expectation values are independent of time $\langle\hat{x}\rangle$, $\langle\hat{p}\rangle$, $\langle\hat{H}\rangle$?

   b. If we make a position measurement and get $x = a/2$ and then *immediately* measure the energy, what values might we get?

   c. Same question, but this time, we measured $x = a/4$ instead.

Quantum Mechanical Harmonic Oscillator

A particle sits in $V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$ such that $\omega \equiv \sqrt{\frac{k}{m}}$. The eigenstates ψ_n can be represented in bra-ket notation as $|n\rangle$

6. Write the position $\hat{x}$ and momentum $\hat{p}$ operators in terms of the raising and lowering operators $\hat{a}_+$ and $\hat{a}_-$.

7. Using these representations determine the following
   a. $\langle n|x|m\rangle$
   b. $\langle n|p|m\rangle$

   For the next two, assume we are *not* talking about the ground state.

   c. $\langle x^2\rangle$
   d. $\langle p^2\rangle$

8. Use the above to prove the quantum analog of the viral theorem $\langle V\rangle = \langle T\rangle = \frac{\langle H\rangle}{2}$