~~In general $|\psi\rangle = (\uparrow_z\downarrow_z - \uparrow$~~

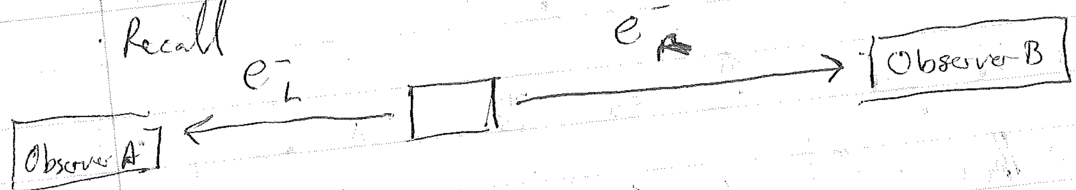
In general $|\psi\rangle = (\uparrow_z\downarrow_z - \downarrow_z\uparrow_z) = e^{i\phi}(\uparrow_\alpha\downarrow_\alpha - \downarrow_\alpha\uparrow_\alpha)$

where $\alpha$ is an arbitrary direction

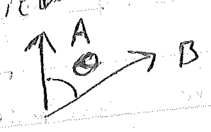
- I measure $e_L$ to give $S_z = \uparrow$ $\therefore$ $e_R$ must be $\downarrow$. But what if instead of measuring $S_z$ on $e_R$ I measure $S_x$ and get $\uparrow$ do I then know 2 components of $e_R$'s spin?
  - Einstein-Podolsky-Rosen "experiment"

2/21 ~~Recall~~ Do Local Hidden Variable Theories make Sense?

- Recall




- An end view




where the axes are the measurement $\measuredangle$'s

- Assume A measures $S_z$
- B measures: $\cos(\theta_{AB}) S_z + \sin(\theta_{AB}) S_x$ ~~(drop~~ $\leftarrow$ dropping $\frac{\hbar}{2}$

$$\cos(\theta)\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} + \sin\theta\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$= \begin{pmatrix}\cos\theta_{AB} & \sin\theta_{AB}\\ \sin\theta_{AB} & -\cos\theta_{AB}\end{pmatrix}$$

- half angles make it easier because rotation by $2\pi \to (-)$

$$\cos(2\phi) = \cos^2\phi - \sin^2\phi$$
$$\sin(2\phi) = 2\cos\phi\sin\phi$$

- let $\theta_{AB} = 2\phi$

- $\cos\theta_{AB} = \cos^2\left(\frac{\theta_{AB}}{2}\right) - \sin^2\left(\frac{\theta_{AB}}{2}\right) = C_{1/2}^2 - S_{1/2}^2$
- $\sin\theta_{AB} = 2\cos\left(\frac{\theta_{AB}}{2}\right)\sin\left(\frac{\theta_{AB}}{2}\right) = 2C_{1/2}S_{1/2}$
- $\hat{S}_B = \begin{pmatrix} C_{1/2}^2 - S_{1/2}^2 & 2C_{1/2}S_{1/2} \\ 2C_{1/2}S_{1/2} & -C_{1/2}^2 + S_{1/2}^2 \end{pmatrix}$
  - eigenvalues are still $\pm$ because still spins
  - → eigenvector for $\lambda = 1$ is $\begin{pmatrix} C_{1/2} \\ S_{1/2} \end{pmatrix}$
  - → since eigenvector for $\lambda = -1$ must be orthogonal the eigenvector is $\begin{pmatrix} S_{1/2} \\ -C_{1/2} \end{pmatrix}$
- If A gets $\uparrow_z$ then B has $\downarrow_z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
  - → then $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = c_1 \begin{pmatrix} C_{1/2} \\ S_{1/2} \end{pmatrix} + c_2 \begin{pmatrix} S_{1/2} \\ -C_{1/2} \end{pmatrix}$
  - → $c_1 = S_{1/2}$, → $c_2 = -C_{1/2}$ } by dot product
  - → B gets $+\hbar/2$ with probability $\sin^2\left(\frac{\theta_{AB}}{2}\right)$
- Q.M predictions for this system:

~~→ N.B. $A \equiv \uparrow_z$~~

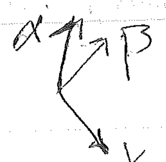
→ $P(\uparrow_A, \downarrow_B) = P(\downarrow_A, \downarrow_B) = \frac{1}{2}\sin^2\left(\frac{\theta_{AB}}{2}\right)$

→ $P(\uparrow_A, \downarrow_B) = P(\downarrow_A, \uparrow_B) = \frac{1}{2}\cos^2\left(\frac{\theta_{AB}}{2}\right)$

~~Hidden Variables~~

## Hidden Varables

- Each $e^-$ has an internal state describing what you get if you measure it
- ~~Need~~ Really $\infty$ # of ~~functions~~ properties one for every spin
- Need at least 3 directions with arbitrary ~~sp~~ $\measuredangle$'s between them call them




- eg $\psi_L = (\uparrow_\alpha, \downarrow_\beta, \downarrow_\gamma)$
  $\psi_R = (\downarrow_\alpha, \uparrow_\beta, \uparrow_\gamma)$
- all possibilities

L

$P_1$ $(\uparrow_\alpha, \uparrow_\beta, \uparrow_\gamma)$
$P_2$ $(\uparrow_\alpha \ \uparrow_\beta \ \downarrow_\gamma)$
$P_3$ $(\uparrow_\alpha \ \downarrow_\beta \ \uparrow_\gamma)$
$P_4$ $(\uparrow_\alpha \ \downarrow_\beta \ \downarrow_\gamma)$
$P_5$ $(\downarrow_\alpha \ \uparrow_\beta \ \uparrow_\gamma)$
$P_6$ $(\downarrow_\alpha \ \uparrow_\beta \ \downarrow_\gamma)$
$P_7$ $(\downarrow_\alpha, \downarrow_\beta \ \uparrow_\gamma)$
$P_8$ $(\downarrow_\alpha \ \downarrow_\beta \ \downarrow_\gamma)$

R

$(\downarrow_\alpha, \downarrow_\beta, \downarrow_\gamma)$
(

opposite

- $\sum P_i = 1$ and $P_i \geq 0 \subset \mathbb{R}$
- Not all $P_i$ = b/c if $\beta \to \alpha$ then $\beta = \alpha$ with higher and higher P as $\beta \to \alpha$

If A measures $\alpha$ and B chooses $\beta$

- Probability that the both get $+\hbar/2$ is
  $\cancel{P_A + P_B}$ $P_3 + P_4$
- Fundamental inequality of probability sense all $P_i \in \mathbb{R}$ and $\geq 0$

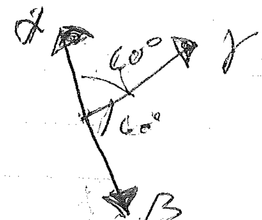
~~i.e.~~ $\cancel{P_3 + P_4 = \sum}$

☆ i.e. $P_3 + P_4 \leq P_3 + P_4 + P_3 + P_7$
$\leq (P_2 + P_4) + (P_3 + P_4)$

→ Probability that if A chooses $\alpha$ and B chooses $\gamma$ both get up is $(P_2 + P_4)$

→ Probability that if A chooses $\gamma$ and B chooses $\beta$ they both get $+\hbar/2$

- For sake of argument




- Redo ☆ in terms of these angles

$$\frac{1}{2}\sin\left(\frac{120^\circ}{2}\right) \leq \frac{1}{2}\sin^2\left(\frac{60^\circ}{2}\right) + \frac{1}{2}\sin^2\left(\frac{60^\circ}{2}\right)$$

$$\frac{3}{8} \leq \frac{1}{4} \leftarrow ???$$

$\therefore$ Hidden ~~vari~~ variables go wrong